NUMERICAL SIMULATION OF A RELATIVISTIC ELECTRON BEAM IN A METALLIC DRIFT TUBE

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1. We will attempt to solve the problem of numerical simulation of the motion of a continuous cylindrical high current relativistic electron beam in an exterior infinite longitudinal magnetic field (motion of a "magnetized" beam) by the "current tube" method. The geometry of the problem can be represented by a closed metallic cylinder (Fig. 1), at the input of which there enters through a foil transparent to electrons an electron beam of radius r_b with uniform density and initial velocity v_{z0}.

The mathematical model consists of a system composed of the Poisson equation and the equations of motion of the relativistic electrons in a vacuum:

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\Phi}{\partial r}+\frac{\partial^2\Phi}{\partial z^2}=-4\pi\rho; \tag{1.1}$$

$$\frac{dP_z}{dt}=\frac{e}{m_0}E_z,\quad \frac{dz}{dt}=v_z,\quad P_z=v_z/\sqrt{1-v_z^2/c^2},\quad E_z=-\frac{\partial\Phi}{\partial z}. \tag{1.2}$$

Boundary conditions for determination of the potential Φ are specified in the form

$$\left.\frac{\partial\Phi}{\partial r}\right|_{r=0}=0,\quad \Phi|_{z=0,L}=\Phi|_{r=R}=0. \tag{1.3}$$

The space charge density ρ needed to close system (1.1)-(1.3) will be determined by the current tube method.

Integration of Eqs. (1.1), (1.2) is performed in a rectangular region

$$D=\left\{-\frac{h_1}{2}\leqslant r\leqslant R,\ 0\leqslant z\leqslant L\right\},\quad \bar{\omega}_{hl}=\{(r_i, z_j),\ i=\overline{0,n},\ j=\overline{0,m}\}$$

with steps in radius – $\{h_1, h_2, \ldots, h_i, \ldots, h_n\}$ and in coordinate z – $\{l_1, l_2, \ldots, l_j, \ldots, l_m\}$.

Equation (1.1) with boundary conditions (1.3) is integrated by the "transverse" drive method [1]. The iteration pattern used has the form

$$\Phi^{\nu+1}-\varkappa\Lambda_1\Phi^{\nu+1}=F^{\nu},\quad F^{\nu}=\Phi^{\nu}+\varkappa\Lambda_2\Phi^{\nu}+\varkappa f, \tag{1.4}$$

where $\varkappa=l_{\min}^2/2$; $l_{\min}=\min\limits_{0\leqslant j\leqslant m} l_j$;

$$\Lambda_1\Phi=\frac{1+\frac{r_{i-1}}{r_i}}{h_i(h_{i+1}+h_i)}\Phi_{i-1,j}-\frac{2}{h_{i+1}h_i}\Phi_{ij}+\frac{1+\frac{r_{i+1}}{r_i}}{h_{i+1}(h_{i+1}+h_i)}\Phi_{i+1,j};$$

$$\Lambda_2\Phi=\frac{2}{l_j(l_{j+1}+l_j)}\Phi_{i,j-1}-\frac{2}{l_{j+1}l_j}\Phi_{ij}+\frac{2}{l_{j+1}(l_{j+1}+l_j)}\Phi_{i,j+1};$$

$\Phi_{0j}=\Phi_{1j}$, $\Phi_{nj}=\Phi_{i0}=\Phi_{im}=0$.

The transition from the ν-th iteration to the $(\nu+1)$-th is achieved by sequential application of the drive method along lines for the three-point Eqs. (1.4).

The electric field intensity E_z is calculated with the following formulas:

$$E_{z_{ij}}=\begin{cases}-\dfrac{4\Phi_{i1}-3\Phi_{i0}-\Phi_{i2}}{2l_1}, & j=0,\quad l_1=l_2,\\ -\dfrac{\Phi_{ij+}-\Phi_{ij-}}{2\min\{l_j, l_{j+1}\}}, & j=\overline{1,m-1},\\ -\dfrac{3\Phi_{im}+\Phi_{i,m-2}-4\Phi_{i,m-1}}{2l_m}, & j=m,\quad l_{m-1}=l_m,\end{cases}$$

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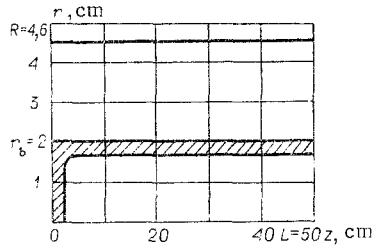

Fig. 1

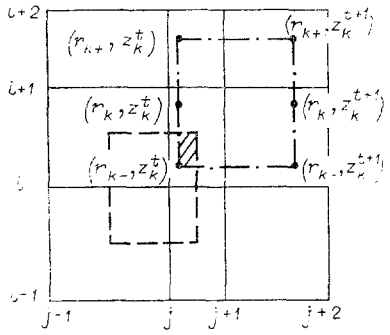

Fig. 2

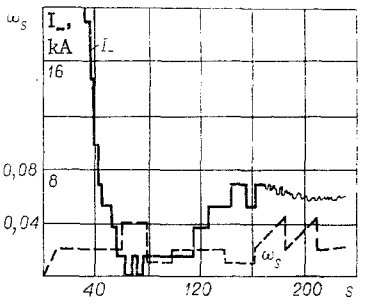

Fig. 3

$$\Phi_{ij+} = \Phi(r_i,\ z_j + l_j),\ \Phi_{ij-} = \Phi(r_i,\ z_j - l_{j+1}),$$

while the values of Φ_{ij+}, Φ_{ij-} at $l_j \neq l_{j+1}$ are determined by interpolation.

The equations of motion are integrated by the Euler method with calculation to second-order accuracy in τ, while $\tau = \Delta T / km = const$ ($\Delta T = L / v_{z0}$, $k \geq 2$ is an integer).

To calculate the space charge density we will employ the method of spreading over areas, the essence of which is the following.

Let (r_k, z_k^t), (r_{k-}, z_k^t), (r_{k+}, z_k^t) (Fig. 2) be the coordinates of the current tube trajectory and the lower and upper limits of the volume of the k-th tube at the moment in time with subscript t; q_k^t is the charge centered in the volume V_k^t, limited by cylindrical surfaces with radii $r = r_{k+}$, $r = r_{k-}$ and the planes $z = z_k^t$, $z = z_k^{t+1}$; V_{kij}^t is the toroidal volume formed by the intersection of the volume V_k^t and the volume V_{ij} generated by the cell network (i, j), i.e., $V_{kij}^t = V_k^t \cap V_{ij}$.

The space charge density at the nodes of the grid is given by the formula

$$\rho_{ij} = \frac{1}{V_{ij}} \sum V_{kij}^t \rho_k^t, \tag{1.5}$$

where $\rho_k^t = q_k^t / V_k^t$. The sum in Eq. (1.5) is taken over all k and t for which $V_{kij}^t \neq 0$.

This method is more economical in comparison to point spreading methods [2] and the method of spreading over areas described in [3], in which the current tube is divided into so-called "incompressible" large particles, having points of the current tube as their coordinates. For solution of the stationary problem of Eqs. (1.1)-(1.3) we use the iteration process of [2]:

$$\nabla^2 \Phi^{s+1} = -4\pi\rho^s,\ \rho^{s+1} = \omega_s \hat{\rho}^{s+1} + (1 - \omega_s)\rho^s,\ s = 0,\ 1,\ 2, \ldots, \tag{1.6}$$

where ω_s is the sequence of relaxation parameters; $\hat{\rho}^{s+1}$ is the space charge density calculated with Eq. (1.5).

The indicated iteration procedure is repeated until the inequality $|\Phi_{ij}^{s+1} - \Phi_{ij}^s| \leq \varepsilon$ is satisfied.

Numerical calculations for currents above the critical value revealed that one may use the establishment of current oscillations at the system output as a criterion of convergence for the iteration process.

For input currents subcritical for the given geometry we choose $\omega_s = 1$. For currents above critical, at $\omega_s = 1$ the process of Eq. (1.6) does not converge, since in various approximations the trajectories first reach the right side of the cylinder, but then circulate. Therefore it is necessary to decrease the space charge density, especially in the first approximations, for which purpose we choose $\omega_s \ll 1$.

2. In the calculations the following initial data was used:

$$R = 4{,}6 \text{ cm},\ r_b = 2 \text{ cm},\ L = 50 \text{ cm},\ v_{z0}/c = 0{,}94,$$
$$n = 28,\ n_b = 21,\ m = 50,$$
$$h_i = \begin{cases} r_b/(n_b - 0{,}5), & i = \overline{1,\ n_b}, \\ (R - r_b)/(n - n_b), & i = \overline{n_b + 1,\ n}. \end{cases}$$

For the initial approximation

$$\rho^0 = I_+ / \pi r_b^2 v_{z0}$$

was chosen.

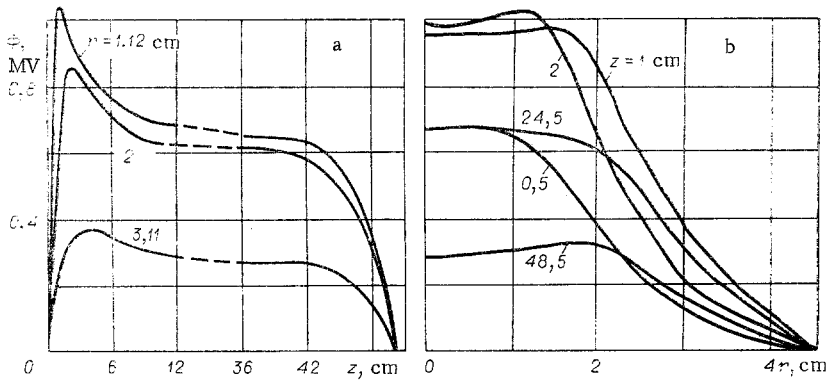

Fig. 4

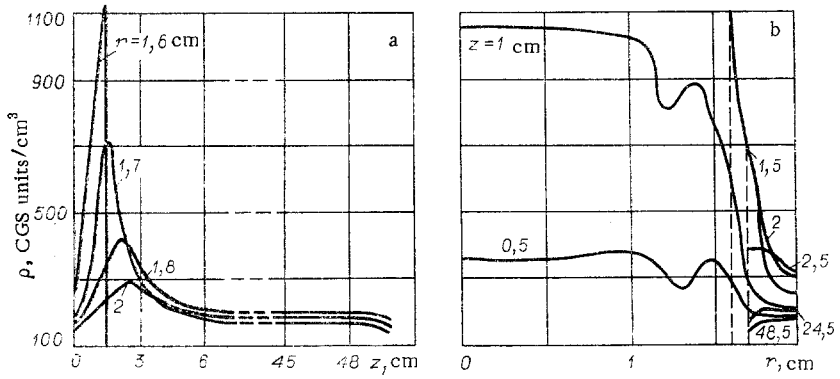

Fig. 5

The convergence of the tube method for currents below critical ($I_+ = 5$ kA) was established in five iterations. For currents above critical ($I_+ = 20$ kA) the form of the electron beam density in the plane (r, z) is shown in Fig. 1. In this case

$$l_j = \begin{cases} 0{,}5L/m, & j = \overline{1,\ m/2}, \\ 1{,}5L/m, & j = \overline{m/2+1,\ m}. \end{cases}$$

A total of 233 iterations of process (1.6) were performed. Beginning with iteration 156, oscillations were observed in the value of the current I_- at the system output (graphs showing I_-, ω_s as functions of the number of iterations s are shown in Fig. 3), the amplitude of which decreases with further calculation, with 5 kA $\leq I_- \leq$ 7 kA.

Experiments performed on the "Tonus" high current relativistic accelerator revealed that at $I_+ = 20$–30 kA with the original metallic cylinder dimensions $I_- = (6 \pm 0.6)$ kA [4]. In connection with this, one can choose established oscillations of I_- with a change in current amplitude of ~5-10% as a criterion for completion of the numerical calculation. In particular, at iteration 233, $I_- = 5.875$ kA.

Graphs characterizing the quantities ρ and Φ for such an output current are shown in Figs. 4 and 5. In the potential Φ we see (Fig. 4a) a sharp increase followed by a gradual decrease to some almost constant value.

It should be noted that in obtaining the desired results the optimal approach was not used, i.e., one must keep in mind that the choice of the sequence of relaxation parameters ω_s was far from optimum, but rather was made to some extent intuitively. A large value of the parameter ω_s leads to an increase in the current amplitude I_-, to abrupt changes in the solution from iteration to iteration (with the exception of the case where the solution is close to the desired one); at low ω_s there is a gradual change in I_-, although this does not permit a conclusion as to the closeness of the intermediate solution to the desired one (this was noted in [5]). All this leads to slowing of the convergence of the iteration process. It is obvious that by using experience in solution of a given problem, one can achieve convergence of the iteration process in a smaller number of iterations by selecting the parameter ω_s in a definite manner [as proved to be the case when an attempt was made to solve the same problem again for the same value of input current (Fig. 6)]. Thus, we may conclude that the value of the relaxation parameter has a large effect on the rate of convergence of the iteration process.

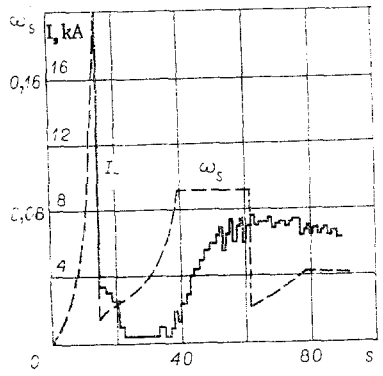

Fig. 6

Moreover, the selection of the initial approximation ρ^0 affects the convergence rate. Therefore in solving similar problems for other values of input current one should use available solutions of the same problem with input currents above critical. Then, considering the form of the relativistic electron beam space charge density function (see Fig. 1), one should choose the tube dimensions to be nonuniform, decreasing them to the limits of the beam.

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